On The Foundations of Computational Photography

Yohann Tendero

Joint work with: Jean-Michel Morel
Bernard Rougé

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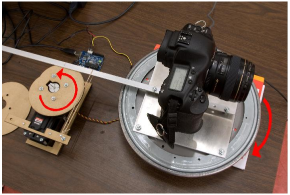
Photographing moving scenes could only be done using short exposure times, until...

Amit Agrawal, one of the inventors of the *flutter shutter* method.

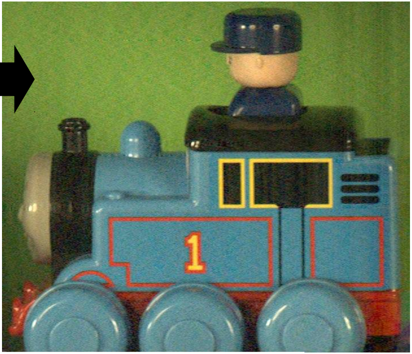
The *flutter shutter* camera.

Anat Levin, one of the inventors of the *motion-invariant photography* method.



The *motion-invariant photography* camera.

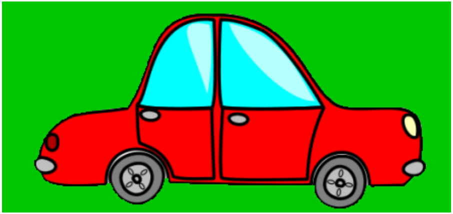
Input Photo



Deblurred Result

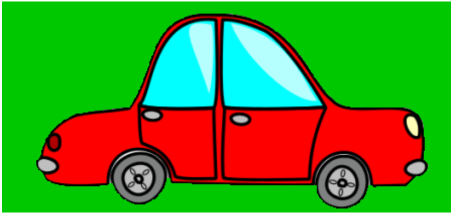
Agrawal *et al.* "Resolving Objects at Higher Resolution from a Single Motion-Blurred Image", *CVPR*, 2007.

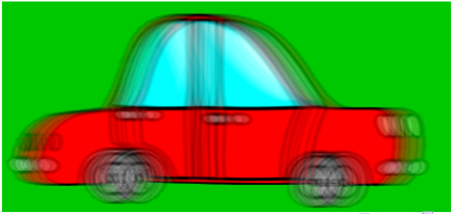
Traditional Camera : Shutter is OPEN

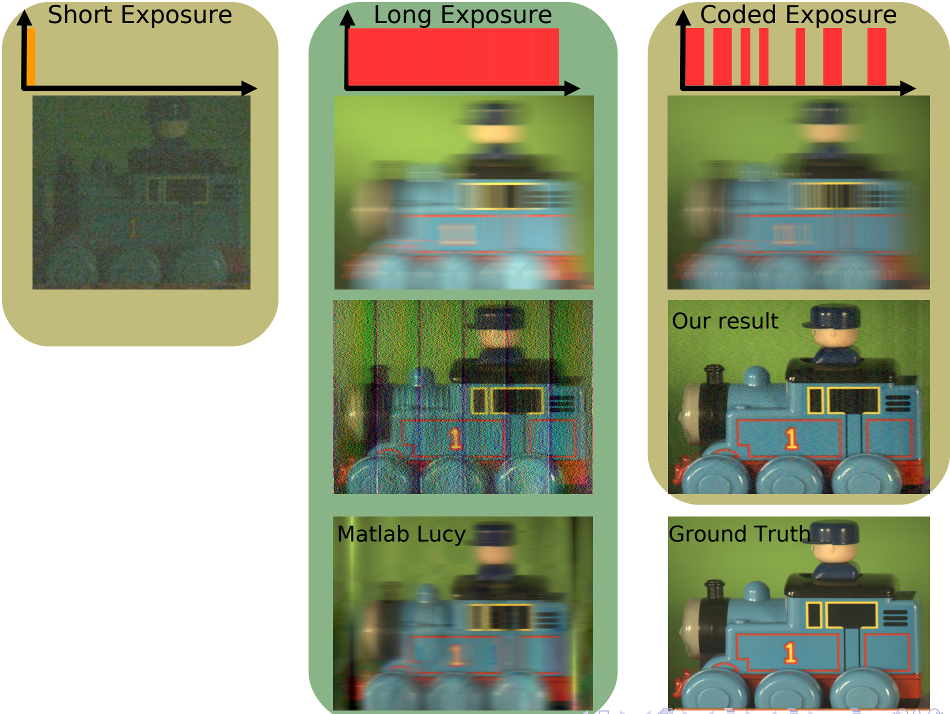


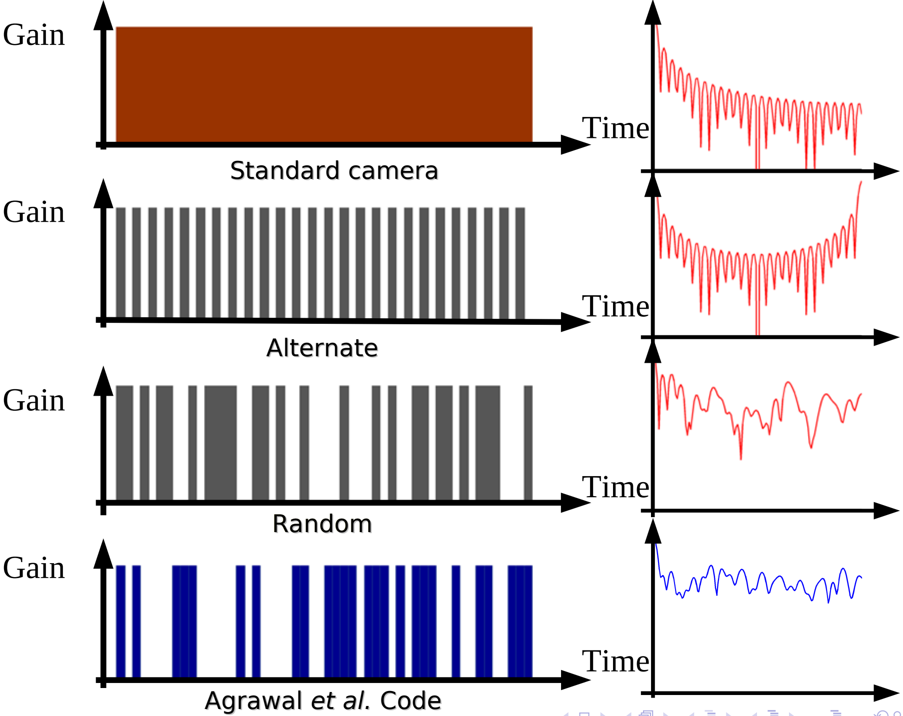
Flutter Shutter Camera : the Shutter

OPENS /CLOSES






Short Exposure
Long Exposure
Coded Exposure
Our result
Matlab Lucy
Ground Truth


Gain
Time
Standard camera
Gain
Time
Alternate
Gain
Time
Random
Gain
Time
Agrawal *et al.* Code

- **Optimal code:** J. Jelinek. "Designing the optimal shutter sequences for the flutter shutter imaging method." 2010.

- **Gain:** The gain (RMSE) of a flutter shutter is bounded above by $\sqrt{1+\frac{\sigma_r^2}{J}}$ and "The gain for computational imaging is significant only when the average signal level J is considerably smaller than the read noise variance σ_r^2" O. Cossairt, M. Gupta, and S.K. Nayar. "When Does Computational Imaging Improve Performance?" 2012.

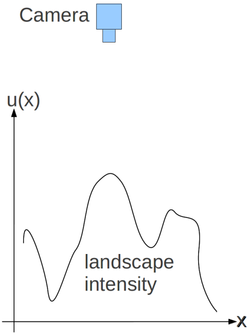
Overview

- What are the *flutter shutter* acquisition formulae?
- Main question: What is the MSE?
- Consequence 1) Optimal *flutter shutter code* for known velocity

  Byproduct: optimal temporal filter for blind motion blur deconvolution
- Consequence 2) Optimal *snapshot* theory
- Consequence 3) Paradox and its solution:

  An optimal (MSE) aperture theory for random velocity models

Image Model



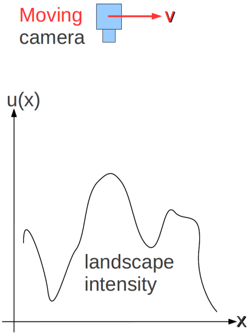
- Δt length of a time interval
- $u = \mathbb{1}_{[-\frac{1}{2},\frac{1}{2}]} * g * I$ ideal observable landscape. Assumption: $[-\pi, \pi]$ band limited and $u \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$

A "Δt snapshot" at a pixel at position n is a Poisson random variable

$$\mathbf{P}_I([0,\Delta t] \times [n - \frac{1}{2}, n + \frac{1}{2}]) \sim \mathcal{P}\left(\int_0^{\Delta t} u(n)dt\right).$$

$X \sim \mathcal{P}(\lambda)$, $\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$.

Image Model



- Δt length of a time interval
- v relative velocity (unit: pixels per second)
- $u = \mathbb{1}_{[-\frac{1}{2},\frac{1}{2}]} * g * I$ ideal observable landscape. Assumption: $[-\pi, \pi]$ band limited and $u \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$

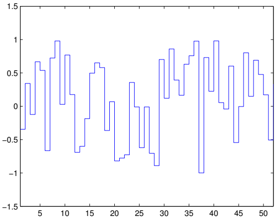
A "Δt snapshot" at a pixel at position n is a Poisson random variable

$$\mathbf{P}_I([0, \Delta t] \times [n - \frac{1}{2}, n + \frac{1}{2}]) \sim \mathcal{P}\left(\int_0^{\Delta t} u(n - vt)dt\right).$$

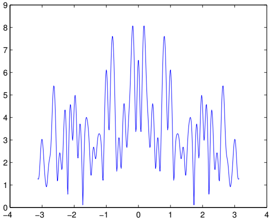
$X \sim \mathcal{P}(\lambda)$, $\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$.

The Numerical *Flutter Shutter* Setup

1. The camera takes a burst of L images using exposure time Δt;
2. The k-th elementary image is assigned a numerical weight $\alpha_k \in \mathbb{R}$;
3. All images are added together to get *one* observed image.



A *non positive flutter shutter function*.

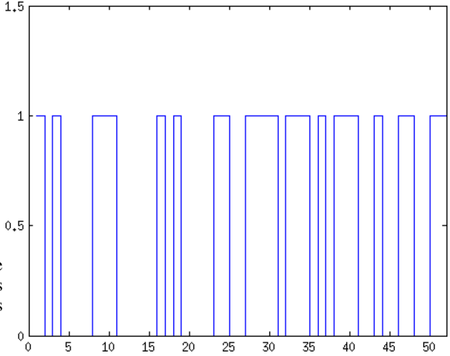


The modulus of its Fourier transform.

3×10^6 candidate codes. We chose a code that (i) maximizes the minimum of the magnitude of the DFT values and (ii) minimizes the variance of the DFT values. The near-optimal code we found is

1010000111000001010000110011110111010111001001100111.

The Agrawal *et al.* code.



The binary *flutter shutter function* for the optimized Agrawal *et al.* code.

$$\text{Code: } (\alpha_0, ..., \alpha_{L-1}) \in \mathbb{R}^L \Leftrightarrow \textit{Flutter shutter function}\text{: } \alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t, (k+1)\Delta t[}(t).$$

Code: $(\alpha_0, ..., \alpha_{L-1}) \in \mathbb{R}^L \Leftrightarrow$ *Flutter shutter function*: $\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$.

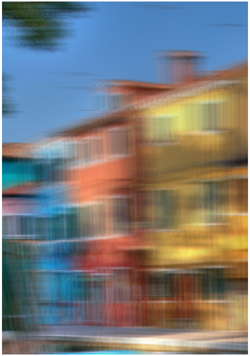
Definition

- *Numerical* samples: $obs(n) \sim \sum_{k=0}^{L-1} \alpha_k \mathcal{P}\left(\int_{k\Delta t}^{(k+1)\Delta t} u(n - vt)dt\right)$.
- *Analog* samples ($\alpha(t) \in [0, 1]$): $obs(n) \sim \mathcal{P}\left(\frac{1}{v}(\alpha(\frac{\cdot}{v}) * u)(n)\right)$.
- Band limited interpolate: $obs(x) \sim \sum_{n\in\mathbb{Z}} obs(n)\mathrm{sinc}(x - n)$.

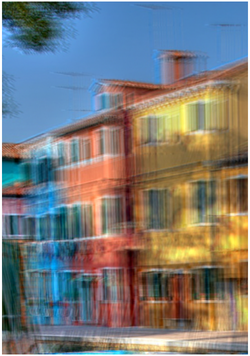
Continuous *numerical flutter shutter* : *any* function $\alpha \in L^2(\mathbb{R})$.

Velocity v *: unit in pixel(s) per* Δt.

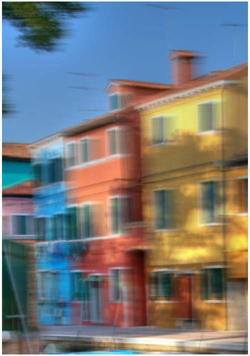
Observed Images Varying the Code



Agrawal *et al.* code.



Random uniform on $[-1, 1]$ code.



The *motion-invariant photography* code.

Inverse Filter Design

Flutter shutter	*Numerical*	*Analog*
Flutter shutter function $\alpha(t)$	$\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$ (with $\alpha_k \in \mathbb{R}$ and $\Delta t > 0$)	$\alpha(t) \in [0, 1]$
$\mathbb{E}\left(obs(n)\right)$ (observed)	$\left(\frac{1}{v}\alpha\left(\frac{\cdot}{v}\right) * u\right)(n)$	$\frac{1}{v}(\alpha\left(\frac{\cdot}{v}\right) * u)(n)$
$var(obs(n))$ (observed)	$\left(\frac{1}{v}\alpha^2\left(\frac{\cdot}{v}\right) * u\right)(n)$	$\frac{1}{v}(\alpha\left(\frac{\cdot}{v}\right) * u)(n)$
Inverse filter $\hat{\gamma}(\xi)$	$\frac{\mathbb{1}_{[-\pi,\pi]}(\xi)}{\hat{\alpha}(\xi v)}$	$\frac{\mathbb{1}_{[-\pi,\pi]}(\xi)}{\hat{\alpha}(\xi v)}$

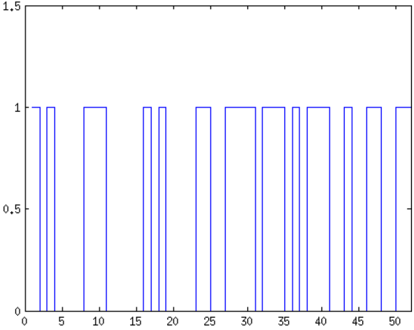
Deconvolved Varying the Code

Code: Agrawal *et al.*.
RMSE $= 2.54$

Code: random uniform on $[-1, 1]$.
RMSE $= 2.25$

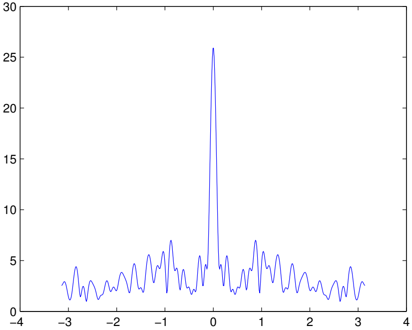
Code: *motion-invariant photography*.
RMSE $= 2.31$

Usual Codes: Agrawal *et al.*



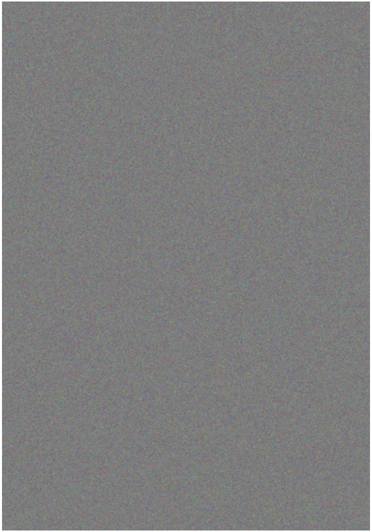
The binary *flutter shutter function* for the optimized Agrawal *et al.* code.

$\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$

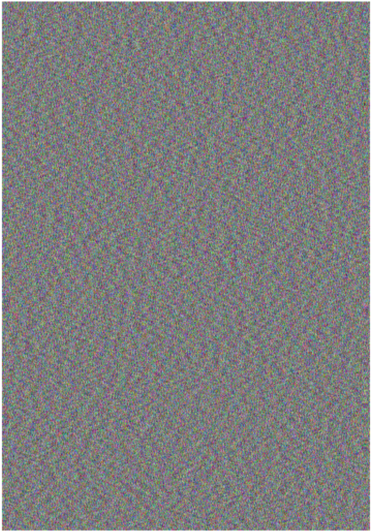


The Fourier transform (modulus) of the *flutter shutter function* with the Agrawal *et al.* code.

$\hat{\alpha}(\xi) = \mathrm{sinc}\left(\frac{\xi\Delta t}{2\pi}\right) e^{\frac{-i\xi\Delta t}{2}} \sum_{k=0}^{L-1} \alpha_k e^{-ik\xi\Delta t}.$



Poisson noise.



Deconvolved Poisson noise using the Agrawal *et al.* code.

Flutter Shutter Formalism Summary

Flutter shutter type	*Numerical*	*Analog*
Flutter shutter function $\alpha(t)$	$\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$ (with $\alpha_k \in \mathbb{R}$ and $\Delta t > 0$)	$\alpha(t) \in [0,1]$
$\mathbb{E}(obs(n))$ (observed)	$\left(\frac{1}{v}\alpha\left(\frac{\cdot}{v}\right) * u\right)(n)$	$\frac{1}{v}(\alpha\left(\frac{\cdot}{v}\right) * u)(n)$
$var(obs(n))$ (observed)	$\left(\frac{1}{v}\alpha^2\left(\frac{\cdot}{v}\right) * u\right)(n)$	$\frac{1}{v}(\alpha\left(\frac{\cdot}{v}\right) * u)(n)$
Inverse filter $\hat{\gamma}(\xi)$	$\frac{\mathbb{1}_{[-\pi,\pi]}(\xi)}{\hat{\alpha}(\xi v)}$	$\frac{\mathbb{1}_{[-\pi,\pi]}(\xi)}{\hat{\alpha}(\xi v)}$
$\mathbb{E}(\hat{\mathbb{u}}_{est}(\xi))$ (deconvolved)	$\hat{u}(\xi)\mathbb{1}_{[-\pi,\pi]}(\xi)$	$\hat{u}(\xi)\mathbb{1}_{[-\pi,\pi]}(\xi)$
$var(\hat{\mathbb{u}}_{est}(\xi))$ (deconvolved)	$\frac{\Vert\alpha\Vert^2_{L^2(\mathbb{R})}\Vert u\Vert_{L^1}}{\lvert\hat{\alpha}(\xi v)\rvert^2}\mathbb{1}_{[-\pi,\pi]}(\xi)$	$\frac{\Vert\alpha\Vert_{L^1}\Vert u\Vert_{L^1}}{\lvert\hat{\alpha}\rvert^2(\xi v)}\mathbb{1}_{[-\pi,\pi]}(\xi)$
MSE	$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\Vert u\Vert_{L^1(\mathbb{R})}\Vert\alpha\Vert_{L^2(\mathbb{R})}}{\lvert\hat{\alpha}(\xi v)\rvert^2}d\xi$	$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\Vert u\Vert_{L^1(\mathbb{R})}\Vert\alpha\Vert_{L^1(\mathbb{R})}}{\lvert\hat{\alpha}(\xi v)\rvert^2}d\xi$

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$\mathbb{E}(\hat{\mathbb{u}}_{est}(\xi))$ (deconvolved)	$\hat{u}(\xi)\mathbb{1}_{[-\pi,\pi]}(\xi)$	$\hat{u}(\xi)\mathbb{1}_{[-\pi,\pi]}(\xi)$
$var(\hat{\mathbb{u}}_{est}(\xi))$ (deconvolved)	$\frac{\|\alpha\|_{L^2}^2\|u\|_{L^1}}{\|\hat{\alpha}(\xi v)\|^2}\mathbb{1}_{[-\pi,\pi]}(\xi)$	$\frac{\|\alpha\|_{L^1}\|u\|_{L^1}}{\|\hat{\alpha}\|^2(\xi v)}\mathbb{1}_{[-\pi,\pi]}(\xi)$
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$var(\hat{\mathbb{u}}_{est}(\xi))$ (deconvolved)	$\frac{\\|\alpha\\|_{L^2}^2 \\|u\\|_{L^1}}{\|\hat{\alpha}(\xi v)\|^2}\mathbb{1}_{[-\pi,\pi]}(\xi)$	$\frac{\\|\alpha\\|_{L^1} \\|u\\|_{L^1}}{\|\hat{\alpha}\|^2(\xi v)}\mathbb{1}_{[-\pi,\pi]}(\xi)$
MSE	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\\|u\\|_{L^1(\mathbb{R})}\\|\alpha\\|_{L^2(\mathbb{R})}}{\|\hat{\alpha}(\xi v)\|^2}\,d\xi$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\\|u\\|_{L^1(\mathbb{R})}\\|\alpha\\|_{L^1(\mathbb{R})}}{\|\hat{\alpha}(\xi v)\|^2}\,d\xi$

Theorem

- *Numerical MSE*$= \frac{\|u\|_{L^1(\mathbb{R})}}{2\pi} \int_{-\pi}^{\pi} \frac{\|\alpha\|^2_{L^2(\mathbb{R})}}{|\hat{\alpha}(\xi v)|^2} d\xi.$
- *Analog MSE*$= \frac{\|u\|_{L^1(\mathbb{R})}}{2\pi} \int_{-\pi}^{\pi} \frac{\|\alpha\|_{L^1(\mathbb{R})}}{|\hat{\alpha}(\xi v)|^2} d\xi.$

Proof: involves a slightly elaborated use of Poisson's summation formula.

Recall: $\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$.

Analog flutter shutter function $\alpha(t) \in [0,1]$.

Every *Flutter Shutter Function* Can Be Made Discrete

Theorem

Let $\beta \in L^2(\mathbb{R})$ *be a continuous flutter shutter function, and* $|v|\Delta t \leqslant 1$

$$\alpha(t) = \sum_{k \in \mathbb{Z}} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$$

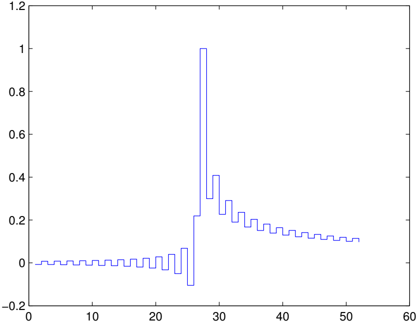
$$\alpha_k = \frac{1}{2\pi} \int_{-\pi|v|\Delta t}^{\pi|v|\Delta t} \frac{\hat{\beta}(\frac{\xi}{\Delta t}) e^{i\frac{\xi}{2}}}{sinc(\frac{\xi}{2\pi})} e^{ik\xi} d\xi.$$

then, $\hat{\alpha}(v\xi) = \hat{\beta}(v\xi)$ *on* $[-\pi, \pi]$.

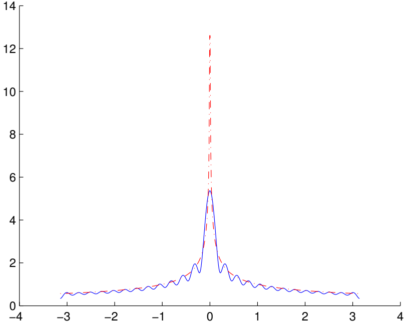
Proof: direct consequence of the Fourier series inversion theorem.

Velocity v: unit in pixel(s) per Δt.

Coded *Motion-Invariant Photography*



The *flutter shutter function* for a *motion invariant photography* code.



Red: Fourier transform (modulus) of the *ideal motion-invariant photography* function.
Blue: Fourier transform (modulus) of the *motion-invariant photography* code, approximating the function in red.

No need of motion direction a priori knowledge.
Avoids physical camera acceleration.

Optimizing *Flutter Shutter*, Patents

- R. Raskar, J. Tumblin, and A. Agrawal. *Method for deblurring images using optimized temporal coding patterns*, 2009. US Patent 7,580,620.
- R. Raskar. *Method and apparatus for deblurring images*, 2010. US Patent 7,756,407.
- S. McCloskey, J. Jelinek, and K.W. Au. *Method and system for determining shutter fluttering sequence*, 2009. US Patent 12/421,296.
- A. Levin, P. Sand, T.S. Cho, F. Durand, and W.T. Freeman. *Method and apparatus for motion invariant imaging*, 2009. US Patent 20,090,244,300.
- ...

Optimal *Flutter Shutter* in Terms of MSE

Theorem

Consider a landscape $u(x - vt)$ moving at velocity v. Then an optimal continuous flutter shutter function minimizing the MSE is equal to $\alpha^*(t) = sinc(tv)$.

MSE$= \frac{\|u\|_{L^1(\mathbb{R})}}{2\pi} \int_{-\pi}^{\pi} \frac{\|\alpha\|^2_{L^2(\mathbb{R})}}{|\hat{\alpha}(\xi v)|^2} d\xi > 0$. (Even though the exposure time is infinite.)

Velocity v: unit in pixel(s) per Δt.

Agrawal *et al.* code, restored image.

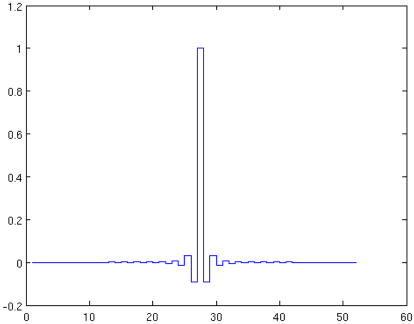
Random uniform on $[-1, 1]$ code, restored image.

The *motion-invariant photography* code, restored image.

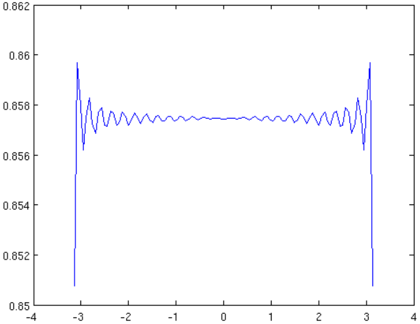
The sinc-code, restored image.

Code type:	Agrawal *et al.*	Random code	M.I.P. code	Sinc code
RMSE	2.54	2.25	2.31	1.46

The sinc-code



The *flutter shutter function* for a sinc-code.



The Fourier transform (modulus) of the sinc-code, approximating the Fourier transform of the ideal gain function.

$$\alpha(t) = \sum_{k=0}^{L-1} \alpha_k \mathbb{1}_{[k\Delta t,(k+1)\Delta t[}(t)$$

$$\hat{\alpha}(\xi) = \mathrm{sinc}\left(\frac{\xi \Delta t}{2\pi}\right) e^{\frac{-i\xi\Delta t}{2}} \sum_{k=0}^{L-1} \alpha_k e^{-ik\xi\Delta t}.$$

The *Flutter Shutter* Paradox in Quotes

- Agrawal *et al.*: "Let us compare to an image captured with an exposure of a single chop, which is equal to T/m seconds. As the **cumulative exposure time** for coded exposure is roughly $T/2$**, MSE is potentially better by** $m/2$ in the blurred region"

- Levin *et al.*: "(about the Agrawal *et al. flutter shutter*) ...the amount of recorded light is halved. Because of the **loss of light**, the vertical budget is reduced from $2T$ to T for each ω_x" and "pends energy outside the slope wedge and thus **does not make a full usage** of the vertical $\hat{k}_{\omega_x}$ budget"

Optimal *Snapshot* in Terms of MSE

Theorem

Consider a landscape $u(x - vt)$ *moving at velocity* v*. Then the optimal aperture time* Δt^* *of a snapshot is designed such that* $|v|\Delta t^* \approx 1.0909$*. Its MSE is*

$$MSE(\Delta t^*) = \frac{v\|u\|_{L^1(\mathbb{R})}}{2\pi} \int_{-\pi}^{\pi} \frac{\xi^2}{\sin^2(\frac{\xi(v\Delta t^*)}{2})} \frac{(v\Delta t^*)}{4} d\xi$$

Velocity v: unit in pixel(s) per Δt.

RMSE comparison of classic *flutter shutters* with respect to the optimal snapshot

Flutter shutter strategy	Gain in terms of RMSE
Optimal snapshot	1
Agrawal *et al.* *flutter shutter* (code) ($v = 1$ $\Delta t = 1$)	0.5636
Ideal motion-invariant photography (infinite time exposure)	0
Motion-invariant photography (with $\left\|\frac{v}{a}\right\| = 1$ and $T = 1$)	0.6233
Ideal flutter shutter (sinc) (infinite time exposure)	1.17

Less than 1 indicates a loss compared to the optimal *snapshot*.

For a known velocity v:

1. optimal *flutter shutter* is derived from a sinc(vt) function
2. optimal *snapshot* satisfies $|v|\Delta t^* \approx 1.0909$
3. the gain of the *flutter shutter* with respect to the optimal *snapshot* is of 1.17 in terms in RMSE
   theory applies to the Levin et al. *motion-invariant photography*
   Y. Tendero, J.-M. Morel and B. Rougé. "The Flutter Shutter Paradox" SIAM Journal on Imaging Sciences
4. this 1.17 bound is also valid when considering sensor readout and obscurity noise (finite variances)
   Y. Tendero and J.-M. Morel. "On the Mathematical Foundations of Computational Photography" UCLA CAM Report 13-65
5. random codes are not the solution
   Y. Tendero. "Are Random Flutter Shutter Codes Good Enough?" UCLA CAM Report 13-84

A Solution to the *Flutter Shutter* Paradox

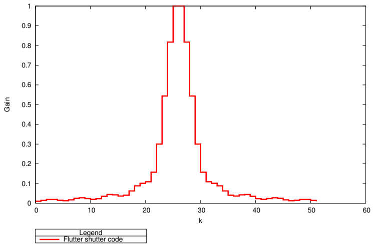
Assume the velocity v comes from a compactly supported probability density $\rho(v)$.

- Forward analysis: formula links $\rho(v)$ and the *flutter shutter code*
- Perform the same optimization for the *snapshot*: optimal exposure time
- Backward analysis: get $\rho(v)$ from a given code

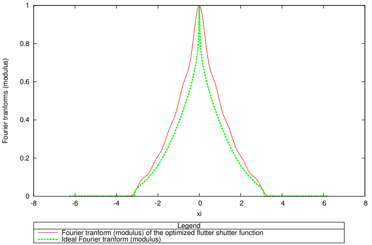
Y. Tendero and J.-M. Morel. "A Theory of Optimal Flutter Shutter For Probabilistic Velocity Models." UCLA CAM Report 13-80.

The gain in terms of RMSE is

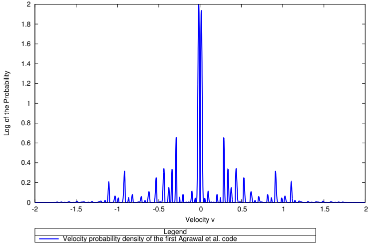
- 5%, when ρ is uniform and exposure time 10 times greater than the optimal snapshot.
- 25%, when ρ is (truncated) Gaussian and exposure time 10 times greater than the optimal snapshot.
- 385%, when $\rho(v) = 0.99\delta_0(v) + 0.01\delta_{15}(v)$ and exposure time 25 times greater than the optimal snapshot.



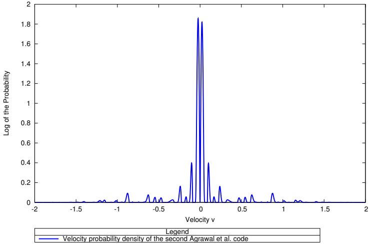
The *flutter shutter function* for a (truncated) Gaussian velocity distribution.



The modulus of its Fourier transform.



The probability density associated with the Agrawal *et al.* code (“Resolving objects at higher resolution from a single motion-blurred image”, *CVPR*, 2007.) : x-axis motion (in signed pixels), y-axis: the logarithm of the velocity distribution ($\log(1+\rho(v))$).



The probability density associated with the second Agrawal *et al.* (“Coded exposure deblurring: Optimized codes for PSF estimation and invertibility”, *CVPR*, 2009.) code: x-axis motion (in signed pixels), y-axis: the logarithm of the velocity distribution ($\log(1+\rho(v))$).

The sinc temporal filter

Beyond pure MSE gain, make camera use more flexible

- The *numerical flutter shutter* is a temporal filter
- The observed is guarantee sharp, as soon as $|v| \leqslant 1$ pixel/frame (sampling theorem).

The sinc *flutter shutter* is the simplest local motion stabilizer.

Images burst $\Rightarrow$ *numerical flutter shutter*: $\sum \alpha_k obs_k$.

(-0.0141, 0.0296, -0.0917, 1.0000, -0.0917, 0.0296, -0.0141, 0.0082)


Before


After
Copyright : Middlebury.


Before


After

Conclusion

1. Provided v is known, the optimal *flutter shutter* is derived from sinc(vt)
2. The optimal *snapshot* has a blur support of approximatively 1.0909 pixel.
3. Gain in terms of RMSE: 17%.
4. Optimize for unknown v, *analytical* formulae linking code and $\rho(v)$ (forward and backward analysis).
5. The theory also provides the optimal exposure time, provided $\rho(v)$.
6. The sinc code permits to perform a blind uniform motion blur deconvolution.