Inverse Problems in Interferometric Phase Imaging

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Phase estimation from interferometric measurements

**Problem:** given a set of observations $e^{j\phi_p} \equiv (\cos \phi_p, \sin \phi_p)$, determine $\phi_p$ (up to a constant) for $p \in \mathcal{V} \equiv \{1, \ldots, n\}$

$e^{j\phi_p}$ is $2\pi$-periodic $\Rightarrow$ nonlinear and ill-posed inverse problem

**Continuous/discrete flavor:**

\[
\phi = \mathcal{W}(\phi) + 2k\pi \quad \mathcal{W} : \mathbb{R} \rightarrow [\pi, \pi]
\]

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**Phase Unwrapping (PU)**

Estimation of $k \in \mathbb{Z}$

**Phase Denoising (PD)**

Estimation of $\mathcal{W}(\phi) \in [\pi, \pi]$ (wrapped phase)
Outline

- Interferometric phase imaging. Examples
- Absolute phase estimation
- Phase unwrapping
- Interferometric phase denoising via sparse regression
- Multisource phase estimation
- Concluding remarks
Applications

- Synthetic aperture radar/sonar
- Magnetic resonance imaging
- 3D surface imaging from structured light
- High dynamic range photography
- Diffraction tomography
- Optical interferometry
- Tomographic phase microscopy
- Doppler echocardiography
- Doppler weather radar
Absolute phase estimation in InSAR (Interferometric SAR)

InSAR Problem: Estimate $\phi_2 - \phi_1$ from signals read by $s_1$ and $s_2$
InSAR Example

Interferogram $\mathcal{W}(\phi_1 - \phi_2)$

Unwrapped phase $\phi_1 - \phi_2$

Geocoded digital elevation model (DEM)

Atacama desert (Chile)

(from [Moreira et al., 13])
Magnetic resonance imaging (MRI)
High dynamic range photography

Intensity camera

Modulo camera

Unwrapped image (tone-mapped)

\[ I_N = \text{mod} \ (I, 2^N) \]
3D surface imaging from structured light (from [Huang et al., 06])

Fringe images

\[ \phi_{r_1} = -120^\circ \quad \phi_{r_2} = 0^\circ \quad \phi_{r_3} = 0^\circ \]

\[ I_k = b_0 + b_1 \cos(\phi - \phi_{r_k}) \]
Forward problem: sensor model

\[ x_i = \cos \phi + n_i \quad \quad n = (n_i, n_q) \]
\[ x_q = \sin \phi + n_q \quad \quad x = (x_i, x_q) \]
\[ n_i, n_q \sim \mathcal{N}(0, \sigma^2) \quad \text{independent} \]
\[ \eta = \mathcal{W}(\phi) + w \]
\[ \mathcal{W}(\phi), w \in [-\pi, \pi[ \]

Data likelihood

\[ p(x|\phi) \propto c e^{\lambda \cos(\phi - \eta)} \]
\[ \eta = \arg(x) \quad \lambda = \frac{2|x|}{\sigma^2} \]
\[ \hat{\phi}_{ML} = \eta + 2k\pi \]
Simulated Interferograms

Images: $\eta = \arg(e^{j\phi} + n)$

$$\text{SNR} = \frac{1}{2\sigma^2}$$
Real interferograms

MRI

InSAR

MRI

InSAR
Bayesian absolute phase estimation

Data term: \( p(\mathbf{x}|\phi) = \prod_{p \in \mathcal{V}} p(x_p|\phi_p) \)

Prior term: \( p(\phi) = \frac{1}{Z} e^{-U(\phi)} \)

Ex: pairwise interactions \( U(\phi) = \sum_{\{p, q\} \in \mathcal{E}} U_{pq} (\phi_p - \phi_q) \)

- \( \mathcal{E} = \{\{p, q\} : p \sim q\} \) clique set
- \( U_{pq} \) clique potential

- \( U_{pq} \) convex
  - Enforces smoothness
- \( U_{pq} \) non-convex
  - Enforces piecewise smoothness (discontinuity preserving)
Estimation criteria

Maximum a posteriori (MAP)  \( \hat{\phi} \in \arg \max_{\phi \in \mathbb{R}^n} p(x|\phi)p(\phi) = \arg \min_{\phi \in \mathbb{R}^n} E(\phi) \)

\[
E(\phi) = \sum_{p \in \mathcal{V}} -\lambda_p \cos(\phi_p - \eta_p) + U(\phi)
\]

\( E \) is hard to optimize due to the sinusoidal data terms

Popular approaches to absolute phase estimation

- Reformulation as linear observations in non-Gaussian noise
- Interferometric phase denoising + phase unwrapping
Phase differences

Wrapped difference of wrapped phases:

\[ \mathcal{W}(\eta_p - \eta_q) = (\phi_q - \phi_q) + (w_p - w_q) + 2\pi l_{p,q} \]

additive noise distributed in \([-2\pi, 2\pi]\]

\[ \eta = \mathcal{W}(\phi) + w \]

wrap errors due to discontinuities, high phase rate, and noise

- In the absence of noise, \( l_{p,q} = 0 \) if \( |\phi_q - \phi_q| < \pi \) (Itoh condition)
- In most applications \( P(|\phi_q - \phi_q| \geq \pi) \) is small but positive
- \( l_{p,q} = 0 \) for \( \{p, q\} \in \mathcal{E} \) if \( \max_{\{p, q\} \in \mathcal{E}} |\phi_p - \phi_q| + \max_{\{p, q\} \in \mathcal{E}} |w_p - w_q| < \pi \)
- Number of wrap errors increases with \( \sigma \). If \( w_p \sim \mathcal{N}(0, \sigma^2) \), then

\[ \mathbb{E}\left[ \max_{\{p, q\} \in \mathcal{E}} |w_p - w_q| \right] \geq \mathbb{E}\left[ \max_{\{p, q\} \in \mathcal{E}} (w_p - w_q) \right] = O\left( \sigma \sqrt{\log |\mathcal{E}|} \right) \]
Absolute phase estimation: linear observations in non-Gaussian noise

\[ y = \mathcal{W}(D\eta) \quad D : \mathbb{R}^n \rightarrow \mathbb{R}^{2n} \text{ – discrete gradient} \]

\[ w_\eta \text{ – interferometric noise} \]

\[ w_\pi \text{ – wrap errors} \]

\[ y = D\phi + w_\eta + w_\pi \]

Histograms of \( y - D\phi = w_\eta + w_\pi \) for a Gaussian phase surface

\[ |\phi_q - \phi_q| < \pi \]

\[ |\phi_q - \phi_q| \geq \pi \]

\( D_h\phi \text{ – histograms for Gaussian and mixture of Gaussians} \)
Formulation based on the linear observation model (LOM)

Minimum $\ell_p$ norm $0 < p < 2$ \cite{Ghiglia & Pritt, 98}

$$\min_{\phi \in \mathbb{R}^n} \| y - D\phi \|_{p,Q} \quad \text{s.t.} \quad A\phi = b$$

Regularized $\ell_1$ norm (convex) \cite{Gonzalez & Jacques, 15}

$$\min_{\phi, u \in \mathbb{R}^n} \| W\phi \|_1 \quad \text{s.t.} \quad \begin{cases} \| y - D(\phi + u) \|_1 \leq \varepsilon_\pi \\ \| u \|_2 \leq \varepsilon_w \\ A\phi = b \end{cases}$$

Adaptive regularized $\ell_2$ norm \cite{Kamilov et al., 15}

$$\min_{\phi \in \mathbb{R}^n} \sum_{i=1}^{n} q_i^t \| y_i - D_i\phi \|_2 + \tau \| H_i\phi \|_* \quad \text{s.t.} \quad A\phi = b$$

$D_i : \mathbb{R}^n \to \mathbb{R}^2$ – discrete gradient

$y_i = (y_{h,i}, y_{v,i})$

Nuclear norm $H_i : \mathbb{R}^n \to \mathbb{R}^4$ – discrete Hessian

Algorithms

IRLS, MM \cite{Lange & Fessler, 95}

PD \cite{Chambolle, Pock, 11}

Seq. of ADMM subproblems $(q_i^t)$

SALSA \cite{Afonso, B-D, Fig., 11}
Example: IRTV ([Kamilov et al., 15]) (SALSA implementation) \( n = 128 \times 128 \)

\[
\max \phi_p = 20\pi \quad \sigma = 0.5
\]

\[
\tau = 10^{-3}
\]

\[
\text{ISNR} = \frac{2n\sigma^2}{\|\hat{\phi} - \phi\|_F^2}
\]

\[
\text{ISNR} = (1.4, 1.5, -16.4) \text{ dB}
\]

\[
\tau = (10^{-4}, 10^{-2}, 10^0)
\]

1 iter (fixed weights)

\[
\text{time} = 20 \text{ s}
\]

10 iters (adaptive weights)

\[
\text{time} = 200 \text{ s}
\]
A few comments on the LOM-based phase estimation

Observation model: \( y = D\phi + w_\eta + w_\pi \)

- self-similar
- highpass
- sparse and \( \phi \)-dependent

- Regularization is challenging. Ex: Tikhonov regularization using \( ||D\phi||^2 \)

\[
\hat{\phi} = \frac{1}{1 + \tau} (\phi + w + D^\dagger w_\pi)
\]

- The wrap errors \( w_\pi \) due to phase discontinuities tend to be sparse and thus well modeled by \( \ell_p \) norms with \( p \leq 1 \)

- \( \ell_1 \) norm (and \( \ell_1 \) on the gradient) yields convex programs but has limited power to cope with wrap errors

1) Denoise (filter out \( w \))

2) (Use \( \ell_p \) with \( p < 1 \)) or (\( p \geq 1 \) and detect the discontinuities)
Interferometric phase denoising + phase unwrapping

Back to MAP estimate

\[ \hat{\phi} \in \arg \min_{\phi \in \mathbb{R}^n} E(\phi) \quad E(\phi) = \sum_{p \in \mathcal{V}} -\lambda_p \cos(\phi_p - \eta_p) + U(\phi) \]

Assume that: \( \phi = \{\phi_p | \phi_p = \eta_p + 2k_p \pi, p \in \mathcal{V}, k_p \in \mathbb{Z}\} \quad (\Leftrightarrow \lambda_p \to \infty) \)

Then:

\[ \hat{k} \in \arg \min_{k \in \mathbb{Z}^n} E(\eta, k) = \arg \min_{k \in \mathbb{Z}^n} U(\eta, k) \]

Integer optimization

Pairwise interactions:

\[ U(\eta, k) = \sum_{\{p, q\} \in \mathcal{E}} V_{pq}(k_p - k_q) \]

\[ V_{pq}(k_p - k_q) = U_{pq}(\eta_p - \eta_q + 2\pi(k_p - k_q)) \]
Phase unwrapping: path following methods

Assume that \( |\phi_p - \phi_q| < \pi \) (Itoh condition)

Then \( \phi_p - \phi_q = \mathcal{W}(\phi_p - \phi_q) = \mathcal{W}(\eta_p - \eta_q) \)

**PU \iff** summing \( \mathcal{W}(\eta_p - \eta_q) \) over walks

\[
\phi_{p_m} = \phi_{p_0} + \sum_{i=1}^{m} \mathcal{W}(\eta_{p_i} - \eta_{p_{i-1}})
\]

Why isn’t PU a trivial problem?

- Discontinuities
- High phase rate
- Noise

\[
|\phi_p - \phi_q| \geq \pi
\]
Phase unwrapping algorithms

- $V_{pq}(\cdot) = \cdot |_{2\pi}$-quantized
  - [Flynn, 97] (exact) sequence of positive cycles on a graph
  - [Costantini, 98] (exact) min-cost flow on a graph \((|\mathcal{V}| = n, |\mathcal{E}| = 4n)\)

- $V_{pq}(\cdot) = (\cdot)^2$
  - [B-D & Leitao, 01] (exact) sequence of positive cycles on a graph \((|\mathcal{V}| = n, |\mathcal{E}| = 4n)\)
  - [Frey et al., 01] (approx) belief propagation on a 1st order MRF

- $V_{pq}(\cdot)$ convex
  - [B-D & Valadao, 07,09,11] (exact) sequence of $K$ min cuts \((KT(n, 6n))\)

- $V_{pq}(\cdot)$ non-convex
  - [Ghiglia, 96] LPN0 (continuous relaxation)
  - [B-D & G. Valadao, 07,09,11] sequence of min cuts \((KT(n, 6n))\)
PUMA (Phase Unwrapping MAx-flow) [B-D & Valadao, 07,09,11]

**Algorithm 1: PUMA**

\[ \phi := \eta, \quad \text{success} == \text{false} \]

**while** \( \text{success} == \text{false} \) **do**

\[ \delta := \arg \min_{x \in \{0,1\}^{\nu}} E(\phi + 2x\pi) \]

**if** \( E(\phi + 2\delta\pi) < E(\phi) \) **then**

\[ \phi := \phi + 2\delta\pi \]

**else**

\[ \text{success} == \text{false} \]

**return** \( \phi \)

---

**Convex priors**

\[ E(k) = \sum V_{pq}(k_p - k_q) \]

- A local minimum is a global minimum
- Takes at most \( K \) (range of \( k \)) iterations
- \( E \) is submodular: \( 2V_{pq}(0) \leq V_{pq}(1) + V_{pq}(-1) \)
  \[ \Rightarrow \) each binary optimization has the complexity of a min cut \( T(n, 6n) \)
PUMA: convex priors

Let $\phi$ be a smooth surface in the Itoh sense. That is $|\phi_p - \phi_q| < \pi$ for $\{p, q\} \in \mathcal{E}$. If $U_{pq}(x)$ is convex and strictly increasing of $|x|$, then

$$\phi = \eta + \hat{k} + c$$

where $\hat{k}$ is the PUMA solution

Related algorithms

[Veksler, 99] (1-jump moves)
[Murota, 03] (steepest descent algorithm for L-convex functions)
[Ishikawa, 03] (MRFs with convex priors)
[Kolmogorov & Shioura, 05,09], [Darbon, 05] (Include unary terms)
[Ahuja, Hochbaum, Orlin, 03] (convex dual network flow problem)
Results

\[ U_{pq}(\cdot) = (\cdot)^2 \]
Results

Convex priors do not preserve discontinuities

\[ U_{pq}(x) = |x| \]

\[ U_{pq}(x) = (x)^2 \]

\[ U_{pq}(x) = \begin{cases} 
  x^2 & |x| \leq \pi \\
  \pi^2 |x/\pi|^{0.5} & |x| > \pi 
\end{cases} \]

\( E_{pq} \) is not graph representable
PUMA: non-convex priors

Ex: \( U_{pq}(x) = \min(x^2, \pi^2) \)

- Models discontinuities
- Models Gaussian noise

Shortcomings
- Local minima are no more global minima
- Energy contains nonsubmodular terms (NP-hard)

Proposed suboptimal solution: majorization minimization applied
PUMA binary sub-problems
Majorizing nonsubmodular terms

Majorization Minimization (MM) [Lange & Fessler, 95]

\[ \begin{aligned}
\tilde{U}(k) &= U(k) \\
\tilde{U}(k + \delta) &\geq U(k + \delta)
\end{aligned} \]

\[ \delta' = \arg\min_{\delta} \tilde{U}(k + \delta) \]

Non-increasing property

\[ U(k + \delta') \leq U(k) \]

Other suboptimal approaches

- Quadratic Pseudo Boolean Optimization (Probing [Boros et al., 2006], Improving [Rother et al., 2007])
- Sequencial Tree-Re Reweighted Message Passing (TRW-S) [Kolmogorov, 2006]
- Dual decomposition (DD) [Komodakis et al., 2011]
- DD + Augmented Lagrangian [Martins et al., 2015]
Results with PUMA (MM)  \((n = 128 \times 128, 2^{nd} \text{ order neighborhood, } p = 0.2, th = 0.1)\)
PUMA/IRTV in a HDRP example \( \phi \in [0, \rho] \quad n = 256 \times 256 \)

PUMA: \( 1^{st} \) order neighborhood, \( p = 0.2 \quad th = 0.1 \)

\[
\phi \mod 2\pi \quad (\rho = 6) \quad \phi \mod 2\pi \quad (\rho = 8)
\]

\[
\begin{array}{c|c|c|c}
\rho & \text{SNR (dB)} & \text{PUMA} & \text{IRTV} \\
\hline
4 & \infty & \infty & \\
5 & \infty & 25.65 & \\
6 & 25.2 & 19.98 & \\
7 & 17.34 & 16.09 & \\
8 & 13.68 & 0.92 & \\
9 & 1.82 & 2.17 & \\
\hline
\text{T(sec)} & 1 & 350 & \\
\end{array}
\]

[Kamilov et al.,15]
Degradation mechanisms: noise + “phase discontinuities”

“Phase wraps”

\[ \frac{|\{ p : |\hat{\phi}_p - \phi_p| \geq \pi \}|}{n} \]

\(\sigma = 0.4\)

\(\sigma = 0.3\)

\(\sigma = 0.6\)

\(\sigma = 1.0\)
Interferometric phase denoising

**objective:** estimate $\mathcal{W}[\phi]$ from $\eta$

phase modulo $2\pi$

original interf. image $\phi_{2\pi} \equiv \mathcal{W}[\phi]$

observed interf. image $\eta$
State-of-the-art in interferometric phase estimation

- Unwrap (first) + denoise
  - CAPE [Valadao & B-D, 09]: unwrap with PUMA and then minimize $E(\phi_\pi, k)$ w.r.t. $\phi_\pi$

- Parametric model for $\phi$
  - PEARLS [B-D et al., 2008]: local first order approximation for phase and adaptive window selection (ICI [Katkovnik et al., 06])

- Denoise $x$
  - WFT [Kemao, 2007]: windowed Fourier thresholding

- Non-local means filtering
  - NL-InSAR/NL-SAR [Deledalle, et al., 11, 15]: patch similarity criterion suitable to SAR images and a weighted maximum likelihood estimation interferogram with weights derived in a data-driven way.
Dictionary based interferometric phase estimation

Motivation

- sparse and redundant representations are at the heart of many state-of-the-art applications namely in image restoration
- phase images exhibit a high level of self-similarity. So they admit sparse representations on suitable dictionaries.

Challenges: the observation mechanism linking the observed phase \( \eta \) with the interferometric phase \( \phi_{2\pi} \) is nonlinear.

Observation: the fact that the amplitude and phase images \( a \) and \( \phi \) are self-similar, implies that \( ae^{i\phi} \) is self-similar.

Our approach: learn sparse representations for \( ae^{i\phi} \) and from them infer \( a \) and \( \phi \).
Interferometric Phase Estimation via Sparse Regression

Complex valued image

\[ x_i = z_i + n_i \in \mathbb{C}^m \]

original vector

noise vector

observed vector

\[ D \equiv [d_1, \ldots, d_k] \in \mathbb{C}^{m \times k} \]
dictionary with respect to which \( z_i \) admits a sparse representation

\[ \hat{z}_i = D\hat{\alpha}_i \quad \min_{\alpha} \| \alpha \|_0, \quad \text{s.t.:} \quad \| D\alpha - x_i \|_2^2 \leq \delta \]
estimation error \( \varepsilon_i = \hat{x}_i - x_i \)
i.i.d. noise \( \Rightarrow \)

\[ \frac{\| \varepsilon_i \|_2^2}{\| n_i \|_2^2} \approx \frac{p}{m} \quad p = \| \hat{\alpha} \|_0 \]
Interferometric phase estimation

\[ P_k \rightarrow \text{the set of patches containing the pixel } k \]

\[ \hat{z}_i = z_i + \varepsilon_i, \quad i \in P_k \]

the set of estimates of \( z_k \) obtained from patches \( i \in P_k \)

Maximum likelihood estimate of \( z_i = ae^{j\phi} \)

(assume that \( \varepsilon_i = [\varepsilon_1, \ldots, \varepsilon_p] \) is \( \mathcal{N}(0, C) \))

\[ \hat{\phi}_{2\pi} = \arg \left( \sum_{j=1}^{q} \hat{z}_j \gamma_j \right) \]

\[ \hat{a} = \frac{\left| \sum_{j=1}^{q} \hat{z}_j \gamma_j \right|}{\left( \sum_{j=1}^{q} \gamma_j \right)} \]

where \( \gamma_j := \sum_{k=1}^{q} [C^{-1}]_{jk} \).

In practice \( \gamma_j \) is very hard to compute and we take \( \gamma_j = c^{te} \).
Dictionary learning

Find a dictionary representing accurately the image patches with the smallest possible number of atoms.

formalization under the regularization framework

\[
\min_{D \in \mathcal{C}, A} L(D, A) \quad L(D, A) = \frac{1}{2} \| X - DA \|_F^2 + \lambda \| A \|_1,
\]

where \( \mathcal{C} := \{ D \in \mathbb{C}^{m \times k} : \| d_j^H d_j \| \leq 1, j = 1, \ldots, k \} \)

and \( X = [x_1, \ldots, x_{Np}] \) and \( A = [\alpha_1, \ldots, \alpha_{Np}] \)

**DL Algorithm:** alternating proximal minimization (APM)

\[
\begin{align*}
D^{k+1} & \in \arg \min_{D \in \mathcal{C}} L(D, A^k) + \lambda \| D - D^k \|_F^2, \\
A^{k+1} & \in \arg \min_A L(D^{k+1}, A) + \lambda \| A - A^k \|_F^2,
\end{align*}
\]

Convergence (based on the Kurdyka-Lojasiewicz inequality)
[Attouch et al. 10], [Xu, Yin, 2012]
Dictionary learning

drawback: alternating proximal minimization takes too long (order of $10^4$ sec) in a typical image scenario ($N_p = 100000$, $m = 100$, and $k = 200$)

Online Dictionary Learning (ODL): [Mairal et al. 2010]

Select randomly $x^t \equiv [x^t_i \ i = 1, \ldots \eta]$ from $z$

(Sparse coding: BPDA)

$\alpha^t := \arg \min_{\alpha \in C^{k \times \eta}} (1/2) \| x^t - D\alpha \|_F^2 + \lambda \|\alpha\|_1$

$\min_{D \in C} \frac{1}{S_t} \sum_{i=1}^{t} w_i \left\{ (1/2) \| x^i - D\alpha^i \|_F^2 + \lambda \|\alpha^i\|_1 \right\}$

$D^t$ converges to the stationary points of

$(1/2) \|X - DA\|_F^2 + \lambda \|A\|_1$, $D \in C$

Computational complexity: $O(km^2 + \eta km)$
The proposed denoising algorithm

**SpInPHASE** [Hongxing, B-D, Katkovnik, 14]

**Input:** $x \in \mathbb{C}^{N_1 \times N_2}$  
**Output:** $\hat{\phi} \in \mathbb{R}^{N_1 \times N_2}$

**Begin**

- $x_i \leftarrow M_i x$, $i = \ldots, N_p$  
  (extract patches)
- $D \leftarrow DL(x_i, i = 1, \ldots, N_p)$  
  (learn the dictionary)
- $\alpha_i \leftarrow OMP(D, x_i, i = 1, \ldots, N_p)$  
  (sparse coding)
- $\hat{z}_i \leftarrow D \alpha_i$, $i = 1, \ldots, N_p$  
  (patch estimate)
- $\hat{x} \leftarrow \text{compose}(\hat{z}_i, i = 1, \ldots, N_p)$  
  (patch compose)
- $\hat{\phi}_{2\pi} \leftarrow \arg(\hat{x})$  
  (interferometric phase estimate)
- $\hat{\phi} \leftarrow \text{PUMA}(\hat{\phi}_{2\pi})$  
  (phase unwrapping)

**End**
DL: Example (truncated Gaussian - $\sigma = 0.3$) $\sqrt{m} = 12, k = 256$

RMSE := $\frac{\|\mathcal{W}(\hat{\phi}_{2\pi} - \phi_{2\pi})\|_F}{\sqrt{N} \mathbf{4N}\pi^2}$

PSNR := $\frac{\|\mathcal{W}(\hat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2}{\|\mathcal{W}(\hat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2}$

$\frac{\|\mathcal{W}(\eta - \phi_{2\pi})\|_F^2}{\|\mathcal{W}(\hat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2} = 20 \simeq \frac{1}{2} \frac{m}{\bar{p}}$

RMSE = 0.048 (rad)
PSNR = 42.35 dB

time (ODL) = 71 sec

time (APS) = 7182 sec
DL: Online (ODL) Versus Batch (APM)

\[ \sqrt{m} = 12, \, k = 512 \]
Restored Images

\( \sigma = 0.5 \)

\( \sigma = 1.0 \)

\( \sigma = 1.5 \)

RMSE = 0.052

RMSE = 0.108

RMSE = 0.174
Results
Dictionary learned from 6 images (shown before)

\[ \sqrt{m} = 12, \ k = 512 \]
Comparisons with competitors

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<th>W</th>
<th>PSNR_a (dB)</th>
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Concluding remarks

- Overview absolute phase estimation, from interferometric measurements, based a linear observation formulation and on phase unwarppping
- The need for interferometric phase estimation
- SpInPhase: Interferometric phase denoising via sparse coding in the complex domain
  - Exploits the self-similarity of the complex valued images
  - State-of-the-art results, namely regarding the preservation of discontinuities coded in the interferometric phase $e^{i\phi}$
- Current research directions
  - Multisource phase estimation
  - Denoising via sparse coding in the complex domain via high-order SVD and nonlocal block matching techniques
  - Phase retrieval with patch-oriented dictionaries
References


References


• V. Kolmogorov and A. Shioura. "New algorithms for convex cost tension problem with application to computer vision." *Discrete Optimization* vol. 6, no. 4, pp., 378-393, 2009.


Multi-source absolute phase estimation

Ex: different frequencies
\[ p_1(z_i|\phi) \propto c_i e^{\lambda_i \cos(f_i \phi - \eta_i)} \]

Two sources. Ex:
\[ f_1 = 1, \quad f_2 = \frac{u}{v} \quad u, v \in \mathbb{N} \quad \text{primes} \]
\[ d(\phi) = -\lambda_1 \cos(\phi - \eta_1) - \lambda_2 \cos(f_2 \phi - \eta_2) \]
\[ d(\phi + 2\pi v) = d(\phi) \Rightarrow 2v\pi\text{-periodic} \]

LOM formulation
\[ \mathbf{y} = \mathbf{D}\phi + \mathbf{w}_\eta + \mathbf{w}_\pi \]
\[ \eta \in \arg \min_{[-v\pi,v\pi]^n} \sum_{p\in\mathcal{V}} d_p(\phi) \]
\[ \mathbf{y} = \mathcal{W}_v(\mathbf{D}\eta) \]

Integer formulation: unwrap phase images with range larger than \(2v\pi\)

\[ \min_{k \in \mathbb{Z}^n} \sum_{\{p,q\} \in \mathcal{E}} V_{pq}(k_p - k_q) \]
\[ V_{pq}(k_p - k_q) = U_{pq}(\eta_p - \eta_q + 2\pi v(k_p - k_q)) \]

Noise is an issue
Example: two sources, image man

\[ W_\pi(x_1) \]

\[ W_\pi(x_2) \]

\[ \rho = 10\pi, \quad f_1 = 1, f_2 = \frac{3}{4} \Rightarrow \nu = 4 \]

SNR = 58 dB
\[ \eta = \arg \min_{\phi} -\lambda_1 \cos(\phi - \eta_1) - \lambda_2 \cos(f_2\phi - \eta_2) \]

\[ f_2 = \frac{2}{3} \]

phase range = 60\pi

SNR = 5 dB
$u/v = 2/3$

range = $60\pi$

$\eta_1$

$v$-PU, $iter = 2$

$v$-PU, $iter = 4$

$v$-PU, $iter = 6$

$v$-PU, $iter = 8$

$v$-PU, $iter = 12$

$T = 2$ sec
v-Interferometric Phase Estimation via DL

**Approach**

\[
\min_{\phi \in [-\pi v, \pi v]} - \log p(z|\phi) + \tau_1 \| e^{j \frac{\phi}{v}} - L(DA) \|_F^2 + \tau_2 \| A \|_{1,1}
\]

**Example**

\[
u/v = 2/3 \quad \text{range} = 60\pi
\]

\[\hat{\phi}_{2\pi v} \ (\text{iter -1 })\]

\[\hat{\phi}_{2\pi v} \ (\text{iter -2 })\]

\[\hat{\phi}_{2\pi v} \ (\text{iter -3 })\]

RMSE \ (\phi_{2\pi v} = 0.86) \quad \text{RMSE} \ (\phi_{2\pi v} = 0.27) \quad \text{RMSE} \ (\phi_{2\pi v} = 0.19)