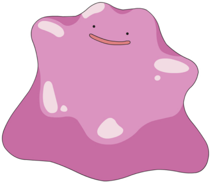
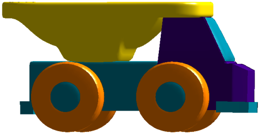
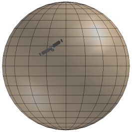
About interpolation on manifolds...





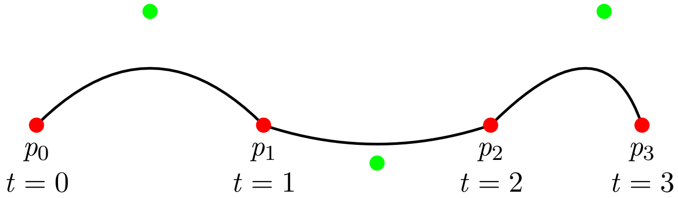


How to interpolate points on curved spaces ?

Light   fast   general   good looking   interpolation

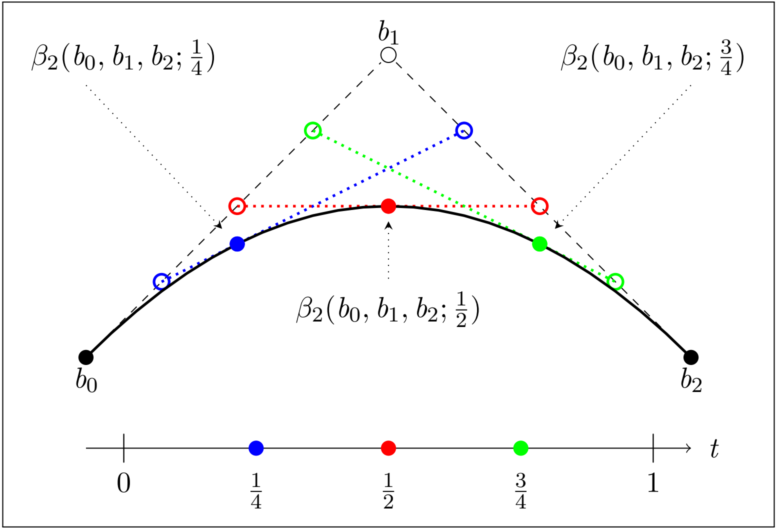
How to interpolate ?

Each segment between two consecutive points is a Bézier function.



Light **fast** **general** **good looking** **interpolation**

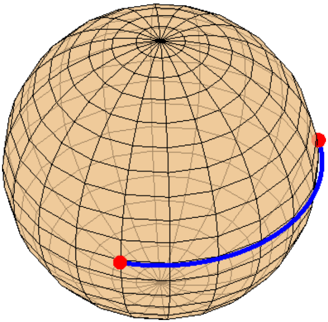
Reconstruction : the De Casteljau algorithm



Light **fast** **general** **good looking** **interpolation**

How to generalize Bézier curves to manifolds ?

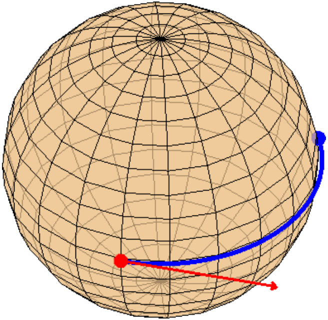
The straight line is a geodesic



How to generalize Bézier curves to manifolds ?

The exponential map to construct the geodesic

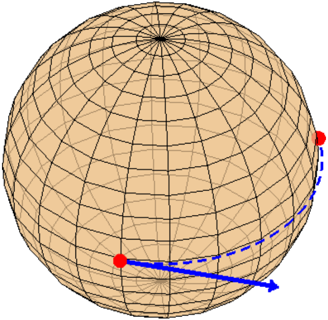
$$\gamma(t) = \mathrm{Exp}_x(t\xi_x)$$



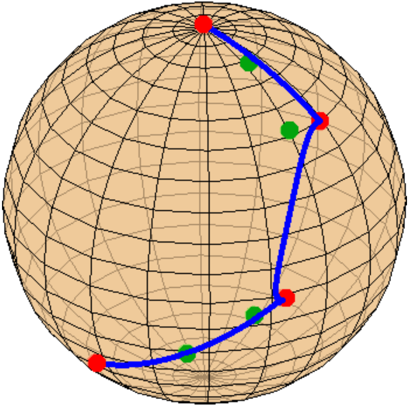
How to generalize Bézier curves to manifolds?

The logarithmic map to determine the starting velocity

$$\mathrm{Log}_{x}(y) = \xi_x$$



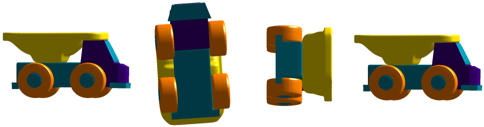
Piecewise interpolation on the sphere



Light **fast** **general** **good looking** **interpolation**

Interpolation on Riemannian manifolds with a $\mathcal{C}^1$ piecewize-Bézier path

Pierre-Yves Gousenbourger



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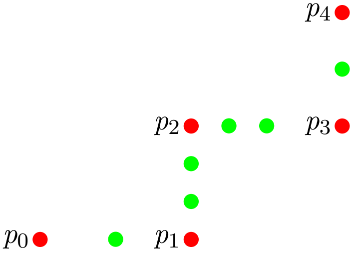
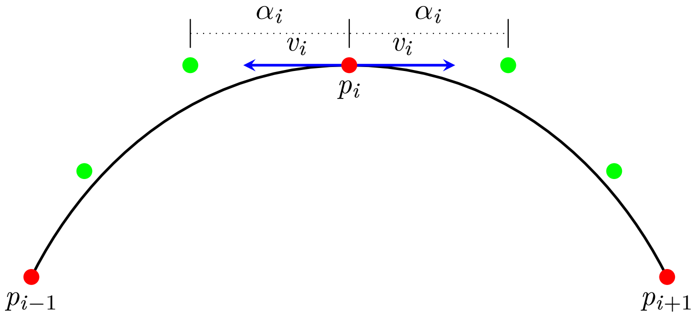
Good-looking curve on the Euclidean space



Find the optimal position of control points

$\mathcal{C}^1$-piecewise Bézier interpolation



$$b_i^L = \mathrm{Exp}_{p_i}(-\alpha_{\mathbf{i}} v_i)$$

$$b_i^R = \mathrm{Exp}_{p_i}(\ \ \alpha_{\mathbf{i}} v_i)$$

Optimal $\mathcal{C}^1$-piecewise Bézier interpolation

Minimization of the mean square acceleration of the path

$$\underbrace{\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_2^0(\alpha_i;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(\alpha_i;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(\alpha_i;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(\alpha_i)}$$

$$\nabla P(\alpha_i)\,!$$

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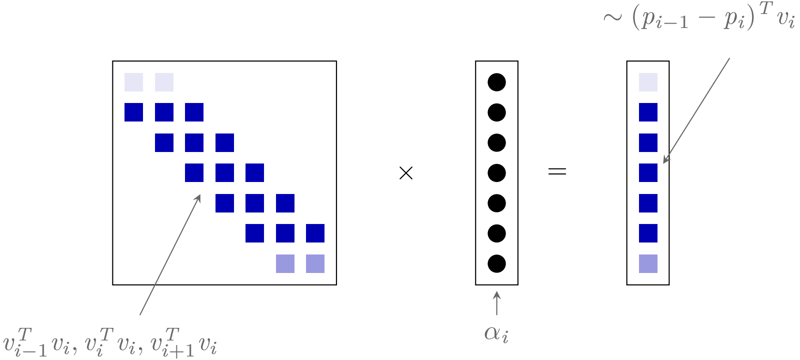

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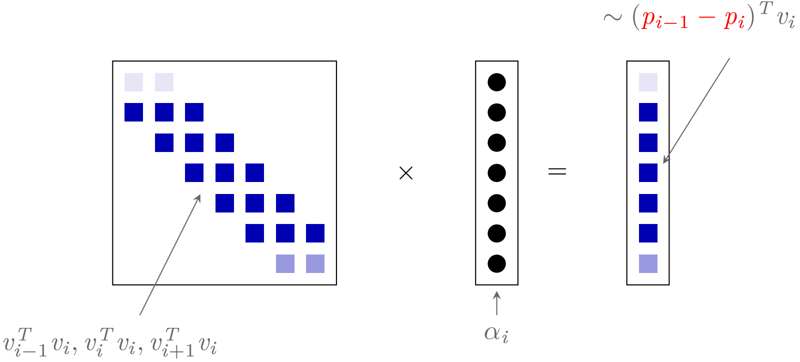

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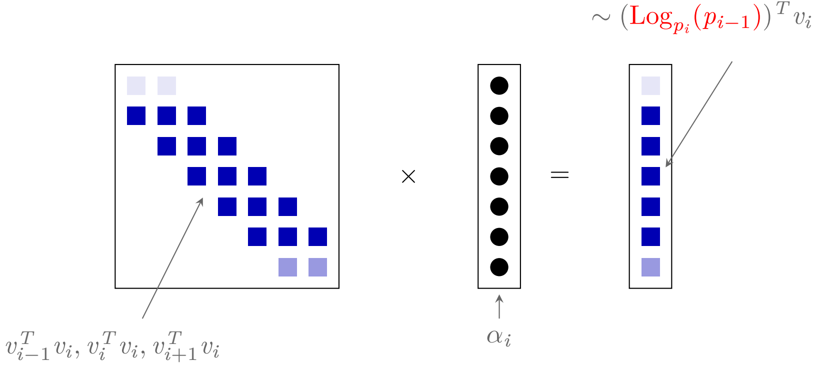

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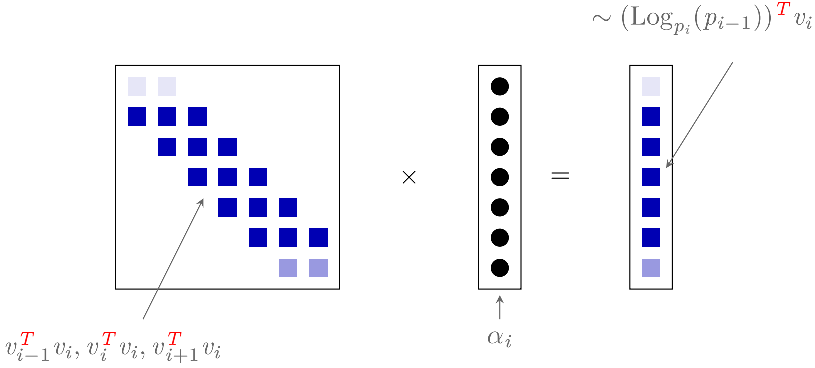

Optimal $\mathcal{C}^1$-piecewise Bézier interpolation

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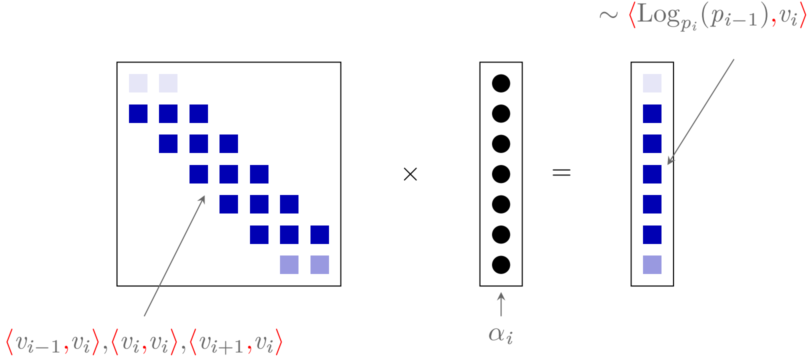
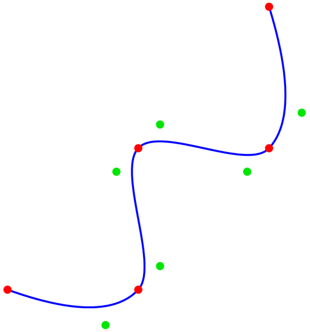
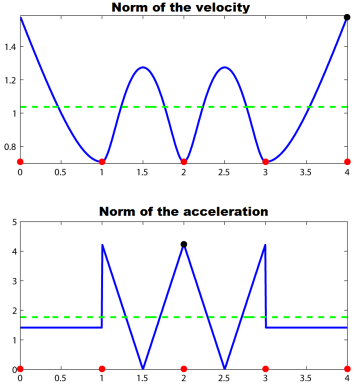
$$\underbrace{\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_2^0(\alpha_i;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(\alpha_i;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(\alpha_i;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(\alpha_i)}$$



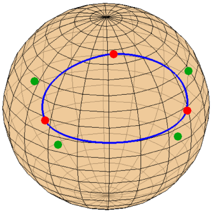
A result on $\mathbb{R}^2$

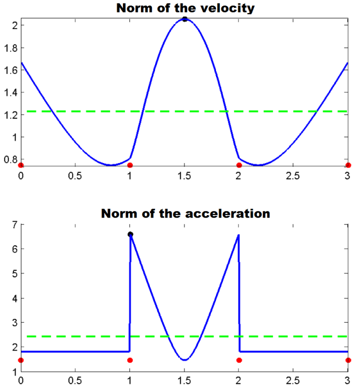




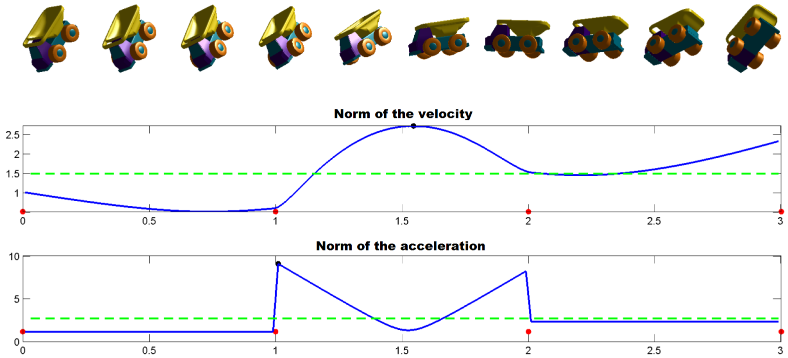
Light fast general good looking interpolation

Generalization to manifolds : the sphere $\mathbb{S}^2$

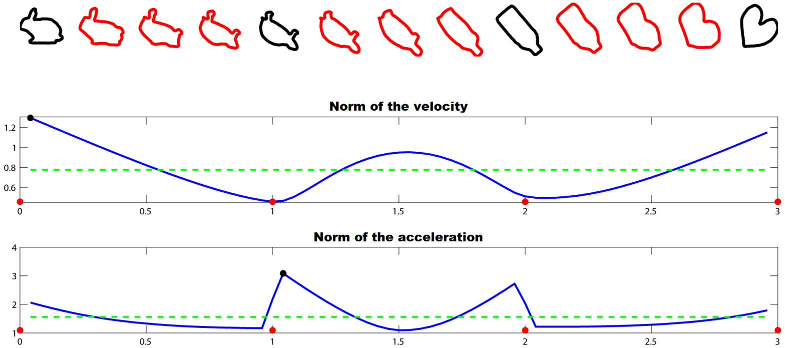




Generalization to manifolds : the special orthogonal group $SO(3)$



Generalization to manifolds : morphing of shapes



Conclusions

Light fast general good looking interpolation

No choice of velocities v_i ? (Arnould, Samir, Absil)
Application to manifolds of high dimension ?

Any questions ?

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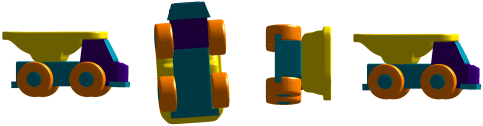

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