A Symbol-Based Bar Code Decoding Algorithm

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One Dimensional Bar codes

- Many different symbologies
- Built-in error correction

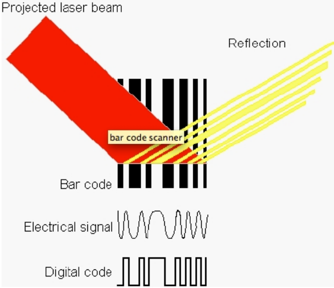
1 23456 78901 2

UPC

123456789012

Code 128

How Bar Codes Are Read



- Laser shines narrow beam back-and-forth on a bar code
- Back scattered intensity converted to voltage signal

Modeling The Measured Signal

- View bar code as a binary function

$$z : \mathbb{R}^+ \to \{0, 1\}$$

- Laser-scanner system acts like point spread function
- Point spread blur depends on scan distance, optics
- Simplest model is Gaussian point spread function

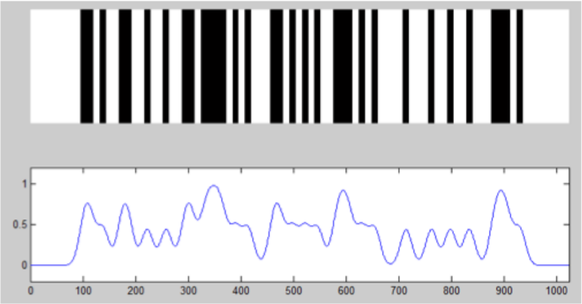
$$g_{\alpha,\sigma}(t) \;=\; \frac{\alpha}{\sqrt{2\pi\sigma^2}} e^{-t^2/2\sigma^2}$$

  where α and σ are unknown
- Signal can be approximated by convolution plus noise

$$h(t) \;=\; \int g_{\alpha,\sigma}(t-s)z(s)ds \;+\; n(t)$$

The Inverse Problem

Existing methods: infer edges between black and white bars "locally" from signal

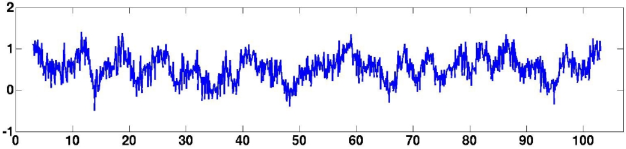


- Theoretical recovery guarantees for small amount of blue and noise in [Esedoglu 04, Esedoglu Santosa, 11]

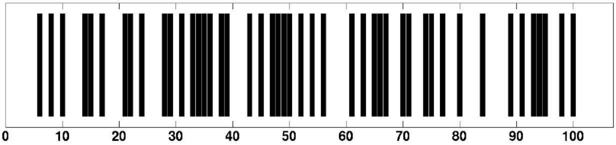
The Inverse Problem

More blur + additive noise

Read noisy data,

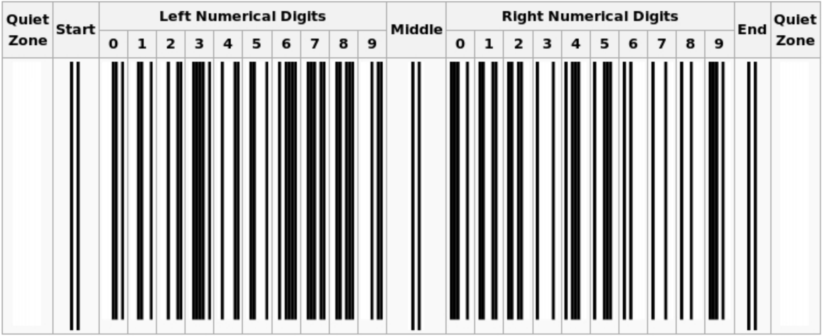


and then try to recover the bar code that produced it.



Universal Product Code (UPC) Bar Codes

- Three types of components: digit, start/end, and middle
- Each digit has a unique sequence of 7 black and white bars
- Fixed start, middle, and end sequences
- Different left and right codes allow upside down reading



Universal Product Code (UPC) Bar Codes

- Each digit can be represented as a binary string of length 7
- Minimum Hamming distance between any two digits is 2

digit	L-pattern	R-pattern
0	0001101	1110010
1	0011001	1100110
2	0010011	1101100
3	0111101	1000010
4	0100011	1011100
5	0110001	1001110
6	0101111	1010000
7	0111011	1000100
8	0110111	1001000
9	0001011	1110100

The UPC Bar Code Dictionary

- Every UPC bar code has the form

$$SL_1L_2L_3L_4L_5L_6MR_1R_2R_3R_4R_5R_6E$$

  where $L_1 \dots L_6$ and $R_1 \dots R_6$ represent UPC code digits
- Take S, $L_1 \dots L_6$, M, $R_1 \dots R_6$, E as columns from matrices

$$L = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&1&1&1&1&0 \\ 0&1&1&1&0&1&0&1&1&0 \\ 1&1&0&1&0&0&1&1&0&1 \\ 1&0&0&1&0&0&1&0&1&0 \\ 0&0&1&0&1&0&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1 \end{bmatrix}, \quad R = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&0&0&0&0&0&0&1 \\ 1&0&0&0&1&0&1&0&0&1 \\ 0&0&1&0&1&1&0&0&1&0 \\ 0&1&1&0&1&1&0&1&0&1 \\ 1&1&0&1&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix},$$

and $\quad S = E = [010]^T, \;\; M = [01010]^T.$

The UPC Bar Code Dictionary

$$\mathcal{D} = \begin{bmatrix} S & 0 & \dots & & & & & & & & & & & \dots & 0 \\ 0 & L & & & & & & & & & & & & & \vdots \\ \vdots & & L & & & & & & & & & & & & \\ & & & L & & & & & & & & & & & \\ & & & & L & & & & & & & & & & \\ & & & & & L & & & & & & & & & \\ & & & & & & L & & & & & & & & \\ & & & & & & & M & & & & & & & \\ & & & & & & & & R & & & & & & \\ & & & & & & & & & R & & & & & \\ & & & & & & & & & & R & & & & \\ & & & & & & & & & & & R & & & \\ & & & & & & & & & & & & R & & \vdots \\ \vdots & & & & & & & & & & & & & R & 0 \\ 0 & \dots & & & & & & & & & & & \dots & 0 & E \end{bmatrix}$$

- The dictionary, $\mathcal{D}$, is block diagonal
- Hamming distance between any two columns is at least 2
- Bar code is $z = \mathcal{D}x$, where $x \in \{0, 1\}^{123}$ has 15 ones

Discrete Representation of Measured Data

Continuous Model

$$h(t) \;=\; \int g_{\alpha,\sigma}(t-s)z(s)ds \;+\; n(t),$$

$$g_{\alpha,\sigma}(t) \;=\; \frac{\alpha}{\sqrt{2\pi\sigma^2}} e^{-t^2/2\sigma^2}$$

- Discretized data is

$$h \;=\; \alpha G_\sigma \mathcal{D} x + n$$

- $x \in \{0,1\}^{123}$ is 15-sparse
- $\mathcal{D} \in \{0,1\}^{95\times 123}$ is the block diagonal dictionary matrix
- $G_\sigma \in \mathbb{R}^{m\times 95}$ blurs the discrete bar code
- Parameters α and σ are unknown
- $n \in \mathbb{R}^m$ is a noise vector

Two Step Solution Approach

Discrete Problem

Recover valid 15-sparse $x \in \{0,1\}^{123}$ given

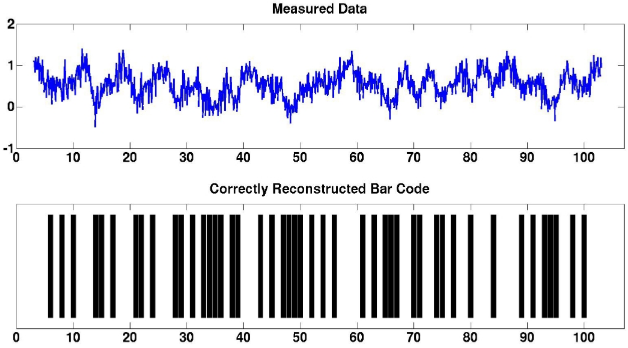
$$h = \alpha G_\sigma \mathcal{D} x + n.$$

- We know that $x_1 = x_{62} = x_{123} = 1$ due to fixed start, middle, and end bar code sequences. Use corresponding blocks of h to estimate α and σ.
- Obtain $\tilde{\alpha} \approx \alpha$ and $\tilde{\sigma} \approx \sigma$
- Greedily recover x digit by digit thereafter by minimizing

$$\left\| h - \tilde{\alpha} G_{\tilde{\sigma}} \left(S L_1 L_2 L_3 L_4 L_5 L_6 M R_1 R_2 R_3 R_4 R_5 R_6 E \right)^T \right\|_1$$

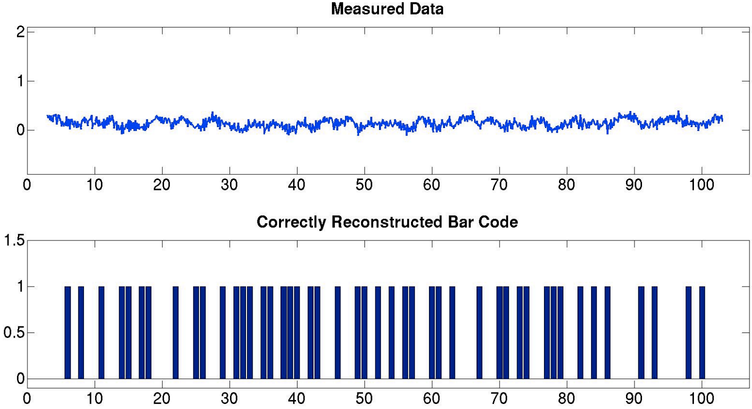
over all 10 choices of L_1 with $L_2 \cdots E$ zeroed out, then all 10 choices of L_2 with $L_3 \cdots E$ zeroed out, etc.

Numerical Example: $\sigma = .75, \alpha = 1$, and 34% Noise



- Noise is i.i.d. Gaussian
- Estimates were $\tilde{\sigma} = 1$ and $\tilde{\alpha} = 1.14$
- Signals decoded correctly about 80% of the time for these noise levels and σ, α settings

Numerical Example: $\sigma = .75, \alpha = .25$, and 10% Noise



Estimates were $\tilde{\sigma} = .8$ and $\tilde{\alpha} = .22$.

What can we prove?

- If blur parameter σ is known, we can show that if the blur is not excessive and the noise level is not too great, the algorithm converges to the right solution.
- We can also show that the reconstruction is insensitive to the value of σ.
- The proof hinges on the matrix

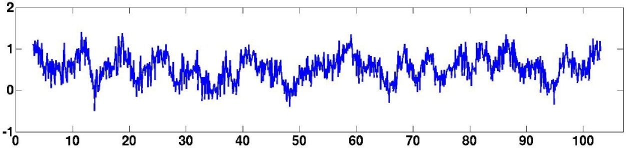
$$G_\sigma \mathcal{D}$$

being "almost block diagonal", and nearly preserving the minimum "hamming distance" between all pairs of columns

Extensions

- Unknown start position
- Variable-length bar codes (Code128)

Read noisy data,



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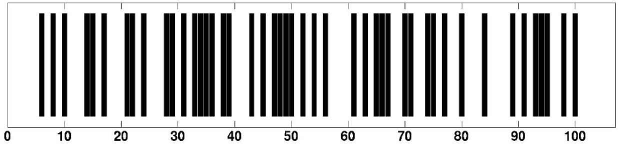

Thank You!

Questions?